# HEINRICH HERTZ AND THE CONCEPT OF FORCE

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HERTZ'S "Principles of Mechanics" contains many passages which have frequently been quoted by philosophers, and his approach to the problems of mechanics has long been regarded as showing a highly philosophical mind.[1] Now Hertz's system of mechanics is of no practical importance; its interest is purely theoretical, and in this article I do not propose to discuss the importance of Hertz's mechanics regarded as a piece of applied mathematics. What I want to do is to discuss Hertz's motives for producing this new system of mechanics. They were somewhat curious, and of considerable philosophical interest. That this is so has not, I think, been sufficiently recognised.

Hertz was dissatisfied with the normal, Newtonian, presentation of dynamics because he thought that some of its fundamental conceptions were extremely unclear. He expressed this dissatisfaction in the following often quoted passage: "Weighty evidence seems to be furnished by the statements which one hears with wearisome frequency, that the nature of force is a mystery, that one of the chief problems of physics is the investigation of the nature of force, and so on. In the same way electricians are continually attacked as to the nature of electricity. Now why is it that people never in this way ask what is the nature of gold, or what is the nature of velocity? Is the nature of gold better known to us than that of electricity, or the nature of velocity better than that of force? Certainly not. I fancy the difference must lie in this. With the terms 'velocity' and 'gold' we connect a large number of relations to other terms; and between all these relations we find no contradictions which offend us. We

are therefore satisfied and ask no further questions. But we have accumulated around the terms 'force' and 'electricity' more relations than can be completely reconciled amongst themselves. We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections that it can be answered; but by removing the contradictions existing between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions." (2)

What are the difficulties about force?* Suppose first of all that we explain the concept of weight, by saying, for example, that two particles joined by a light string which passes over a pulley have the same weight if they balance, that a mass A which balances two masses B and C has a weight equal to the sum of the weights of B and C, that a spring which balances a weight A has a tension equal to the weight of A. Here we have a statical conception of force. We might extend the concept of force by considering unbalanced forces. We should then find that an unbalanced force $F$ will give a particle of mass $m$ an acceleration $f$, where $F = mf$. This is Newton's second law of motion, and looked at in this way the law is evidently an empirical one, an induction from experience of the various accelerations given to bodies by stretched springs and so on. On the other hand, this account is a complete distortion of the concept of force

* These are brought out to some extent on the first half of p. 12 of the Introduction. Hertz also refers to objections of a slightly different nature, connected with the question of the *simplicity* of the Newtonian representation. See especially p. 13 of the Introduction. These are not altogether difficulties in respect of the *clarity* of the representation, and I do not discuss them here, but in fairness to Hertz I suggest that the reader should not on this account omit them from his consideration.

as it occurs in dynamics. For there the second law of motion must be understood in conjunction with the first and third laws. Consider the first law. This says that a body which is not acted on by a force moves with uniform velocity in a straight line. If, however, a body, for example a stone of mass $m$, is thrown into the air and is observed to move in a curve, we say that it must be acted on by a force, and that if it has an acceleration $f$ in a certain direction, then there must be a force $mf$ acting on it in that direction. It now looks, however, as though we have reduced the first and second laws of motion to tautologies. Is "$F = mf$" as it occurs in Newtonian dynamics a *definition* of force? This would reduce the first and second laws of motion to trivialities. Shall we agree that they are, and that the factual weight of the theory rests on the third law? The theory would then state that if we choose suitable axes of reference we can find "a certain decomposition of the rates of change of momenta, relative to the axes, of all particles of the universe, namely one in which the components occur in pairs, the members of each pair belonging to two different particles and being opposite in direction, in the line joining the particles, and equal in magnitude. Each component in the proposed decomposition is called a force and is said to act on the particle to which it belongs." [3] Here the observables are distances and velocities: forces and masses are defined in terms of these. It is a beautiful exposition of the situation as it confronts us in astronomy. But what of the connection between "mass" here and mass as measured in the laboratory, by balance? And do Newton's laws of motion reduce to the assertion that a certain programme *can* be carried out (that axes *can* be chosen so that a certain decomposition *can* be made)? What does "can" mean? It does not mean that the programme *has* everywhere been carried out, but nor is it merely a pious hope. It means that a certain amount of success has been achieved in the past; (of course the success has been, in fact, enormous; consider for example the discovery of Neptune, and the application of Newton's laws to the motion of double stars, and even,

to some extent, to the rotation of the galaxy) : it also means that continued success is to be expected in the immediate future. But all this is very vague. Is the concept of force as vague as this?

Let us look at the same thing from a slightly different angle. Suppose we throw a ball into the air. It moves in a curve, though there is no force, in the statical sense of "force", acting on it. We say that there is a gravitational force. Is this statement a triviality? No, for we would only make it if there was (as there is) a reasonably simple law of gravity such as the inverse square law. If "gravitational" attraction were so haphazard that no law of gravitation seemed to exist we should not speak of gravitational force, for the assertion that there was a gravitational force on a body with an otherwise unaccountable acceleration would be trivial. It would not enable us to connect up the acceleration of the body in these circumstances with its acceleration in other circumstances, or to perform any calculations to predict its subsequent motion. The first and second laws of motion would then indeed be trivial tautologies. Again, when an iron ball is allowed to drop past a powerful magnet or an electrified pith ball is allowed to fall past a similar ball, the ball has an acceleration in addition to that predicted by the law of gravity, and so we speak of a magnetic or electrostatic force. This would be trivial if there were no reasonably simple law of magnetic or electrostatic attraction discoverable. In fact, as we all know, there are reasonably simple laws of magnetic and electrostatic attraction. In other words, to save the Newtonian laws by regarding them as defining "force" can indeed be done, but when we do it we always "take the responsibility of opening up a new branch of physics",[4] and hence we, so to speak, bet that our predictive methods can be successfully extended. So though we may treat the Newtonian laws as if they were definitions of "force" we are nevertheless making a bet, we are not treating them merely as definitions. All this is what makes the question "are the laws of motion analytic or empirical?" such a misleading question. We feel

like answering that they are both and neither. The criteria for asserting them are in the main well known, but their number is not definite, and sometimes we must (to use the language of military strategists) judge of imponderables. We have rules for the use of the word "force", but not strict rules. Our worry occurs because we want to fit the theoretical laws of dynamics into one or other of the logician's pigeonholes "analytic" and "synthetic", and they aren't made for either. (Nor are they *obviously* made for *neither*.)

Hertz contrasts "force" and "electricity" with "velocity" and "gold". He says we feel confused about the former pair of concepts in a way in which we do not feel confused about the latter pair, but does not put his finger very clearly on why we are confused in the one case and not in the other. One important difference seems to be this: the questions "what is velocity?" and "what is gold?" are in certain circumstances intelligible. Whereas "what is force?" and "what is electricity?" are not normally intelligible. "What is velocity?" may be interpreted as a request for a definition of "velocity", and the concept of velocity is one of the comparatively restricted set of concepts with respect to which it makes sense to ask for a definition. "Velocity" is a shorthand concept, just as "oculist" is. "Oculist" is short for "eye-doctor". Again, "googly" is a shorthand word. It is shorthand for "off-break delivered with a leg-break action". But most words are not shorthand words, and it is obvious that not all words can be. "Velocity" is shorthand for "$\underset{dt \to 0}{Lt} \frac{ds}{dt}$" where $s$ is the distance travelled in time $t$. On the other hand "what is gold?" is not interpretable as a request for a definition of "gold", for "gold" is not a shorthand word.[5] But in certain circumstances "what is gold?" is intelligible; it may be answered by pointing, "Look, that and that and that are gold". Or, again, we could say "gold is a malleable, yellow, shiny metal", which though not acceptable as a *definition* of "gold", would go some way as part of an *explanation* of the meaning of "gold".

There is an important class of words that are not shorthand words. These are words like "blue", "sweet", "shrill". To explain the use of the word "blue" to a blind man is impossible; he has no use for it. We explain "blue" ostensively—we point to blue things and teach a child to pick blue things from a heap when we say "blue", red things when we say "red", and so on. Let us call such concepts "ostensive concepts". Now many philosophers have called ostensive concepts "indefinable", and have supposed that all concepts are either indefinable or definable. This is to make most of our concepts shorthand ones. This, however, is clearly wrong. For many concepts can be explained linguistically, non-ostensively, but the rules for their use are not strict, and so the linguistic explanation cannot take the form of a definition. We saw that "force" is of this sort. So again is "electricity". So again is "length". We can explain how to measure length, but there is no definition of "length", for length is not a shorthand concept. Nor again is "nation". We can explain what is meant by "nations" by giving sample sentences about people which would be held to verify or confute a given statement about nations, but, as is well known, there are no strict rules of translation. The dichotomy "definable-indefinable" is thus another of logic's bogus dichotomies. Part of the trouble is as follows. The dichotomy "definable-indefinable" works perfectly in mathematics. In pure mathematics some of the concepts are indefinable in the sense that they are "implicitly defined" by the axioms: lines, points, planes, etc. in an axiomatic treatment of geometry, for example, are just regarded as entities which satisfy the axioms, and various "interpretations" are of course possible. All other concepts are shorthand concepts. Thus "circle" is shorthand for "curve every point of which lies at the same distance from a given point". It would not affect the mathematics but only time and ink and temper, if everywhere "circle" were struck out and replaced by the longer phrase. Taking pure mathematics as our model, we easily tend to ask for definitions of the various concepts of empirical science and of ordinary life.

But language outside mathematics has an empirical job to do, and an important feature of language that has an empirical job to do is that it necessarily takes a form in which the dichotomy "definable-indefinable" is ludicrously inappropriate. This is so not least in science. Who ever saw a definition of "electron"? It would be impossible to give the use of the word "electron" by giving a hard and fast rule for its use. But the use of the word "electron" can be given an explanation, though not a strict explanation. How would one explain to a person the use of the word "electron"? One would describe cathode ray tubes, the Wilson cloud chamber, the experiment to determine the ratio of the charge to the mass of the electron, and so on. Later one would explain the rôle "electron" plays in theory, in wave mechanics, say. Gradually the person would become more and more competent in the use of the word "electron". But we would not have given him a definite set of rules. He would have to use his common sense. Thus if wave mechanics were scrapped, or if a particular experiment were discredited, he would not now say that the entities in question were not electrons. Provided a reasonable proportion of the criteria remained he would continue to speak of electrons. There is no short cut to acquiring the competence which is the ability to use the word "electron" intelligently; "electron" is not a shorthand concept. Of course there are shorthand concepts in physics, like "work", which is short for "integral of force with respect to space", but the interesting concepts are not shorthand ones.

Similarly in ethics the interesting concepts are not shorthand ones, but nor are they ostensive ones like "yellow". We can give a linguistic explanation of the use of "good" but not a strict one. There is no *definition* of "good"; there is no definite set of rules for its use. Nevertheless there is an indefinite set of rules for its use. A child can say "what is a bad man?" and as a first approximation his mother can say "a bad man is a man who tells lies, or who is cruel, or who kills people, or who breaks promises, and so on". Again the child can point to a man who is putting his whiskers in his

soup. "Bad man", he will say. "No, not bad", his mother will reply, "a man who steals is bad, but a man who puts his whiskers in the soup is not bad, he is rude". Gradually the child will learn how to apply the words "good", "bad", "polite", "rude" and so on correctly. He will have acquired a linguistic competence. On the other hand, it is not a competence which can be inculcated by inculcating a strict rule. It is more like the competence of a batsman at stroke-playing than the competence of the umpire at counting the number of balls in an over. It is small wonder, then, that the uncritical taking over from mathematics, where it works, to ordinary, scientific, or ethical language, where it does not and could not work, of the definable-indefinable dichotomy leads to trouble in ethics as it does elsewhere. For example, in *The Foundations of Ethics,* p. 5, Sir David Ross says "The question what is the relation between the attributes goodness and rightness is, however, only part of a larger question or series of questions which can be asked about either one of them. About each of them we can, to begin with, ask the question whether it is definable or indefinable". He seems never to have considered the possibility that they are neither. The whole book takes its course from a fundamentally wrong start.

If "N" is a shorthand word, then an expression of the form "N is . . ." will normally be taken as giving the *definition* of "N". On the other hand, if "N" is not a shorthand word there will be no such meaningful sentence of the form "N is . . .". Thus "electricity" is not a shorthand concept, and so there is no meaningful sentence of the form "electricity is . . ." in the way that "an oculist is . . ." can meaningfully be completed by "an eye-doctor" or "a chimney sweep", yielding a correct or incorrect statement. (Of course there are statements *about* electricity, such as "electricity is generated at Loch Sloy", but there is no temptation to regard them as statements "about the nature of electricity".) "Electricity is . . ." is a bad beginning for a statement, just as "virtue runs . . ." is.

N. R. Campbell saw this point very well. In a passage strikingly reminiscent of the one from Hertz, but in many ways far clearer, he says: "There are few of us who have not been asked, and considerably puzzled, by a familiar question from unscientific acquaintances, What is electricity? The most accomplished physicists are seldom ready with an answer; the right answer is that the question is unanswerable because it does not mean anything at all. The question is suggested only by the idea that sentences are capable of being analysed into constituents each of which has a separate significance, and, in particular, that any word which is grammatically a noun is capable of being the subject of some significant statement in the subject-predicate form. Our friend has heard statements made in which the word *Electricity* is used; and he jumps to the conclusion that this word can be extracted from the sentence, placed in another beginning 'Electricity is . . .', and that some sentence of this form must have a clear meaning." (6)

The concept of "force" loses its puzzling character once we don't try inappropriately to fit it into one of the definable-indefinable pigeonholes, and once we see the nature of the Newtonian laws, and how they, too, have been puzzling, because philosophers have tried to force them into inappropriate pigeonholes, this time into one or other of the analytic-synthetic pair. Hertz, on the other hand, thought(7) that the people who *used* the concept (as opposed to those who discussed it) were confused. He therefore constructed a mechanics in which the concept did not essentially occur. Though his system is intrinsically interesting, and I do not wish to question the value of his achievement, his chief motive was bad. There is no reason to suppose that the people who *use* the concept "force" are confused; the concept only appears confused when we are confused in *discussing* it, when we ask "is force definable or indefinable?" or "into which of the hitherto recognised logicians' pigeonholes do the laws of motion go?" Indeed there is scarcely any concept in physics (apart from the shorthand ones) about which philosophers

have not been impelled at one time or another to ask inappropriate questions, e.g. "what is Length?" "what is the real length of the metre rod?" "what is Time?" and so on. Hertz gets rid of the concept of force but uses concepts like position and distance about which the inappropriate form of question could just as easily be asked. Philosophical troubles will arise with these concepts, just as with those of "force" and "electricity", if we persist in asking inappropriate questions. It is not the physicists' concepts that have been confused; it has been the logicians who discussed them. But their confusion has led to illumination; in particular it has led us to see how language within mathematics (which has no truck with anything empirical) differs from language outside. Outside mathematics we are usually concerned with eventualities; since we cannot foresee eventualities our linguistic rules leave a region of indefiniteness.

*References.*

(1) Cf. Russell, *Principles of Mathematics*, p. 495: "This system (i.e. Hertz's) . . . far simpler and more philosophical in form than Newton's."

(2) *The Principles of Mechanics*, translated by D. E. Jones and J. T. Walley, pp. 7-8.

(3) The quoted passage comes from *Newton's Theory of Kinetics* by W. H. Macaulay. Bulletin of the American Mathematical Society, Vol. III, 1896-7. This is the clearest exposition of the Newtonian theory that I know of; my attention was drawn to it by Dr. S. E. Toulmin.

(4) I owe this illuminating phrase to Dr. S. E. Toulmin.

(5) I have discussed this matter in some detail in an article "Descartes and the Wax", *Philosophical Quarterly*, Vol. I, No. 1.

(6) *Physics, The Elements*, p. 53.

(7) Though with reservations. See *The Principles of Mechanics*, pp. 8-9.